DTIC FILE COPY

AFOSR-TR- 88-1029

(2)

FINAL REPORT: PART 2

CONSTITUTIVE MODELLING OF JOINTS UNDER CYCLIC LOADING

AD-A200 233

PART 2: FURTHER DEVELOPMENT OF HIERARCHICAL PLASTICITY MODEL FOR JOINTS

Submitted to

Air Force Office of Scientific Research (AFOSR)

Bolling AFB, D. C. 20332

Grant No. AFOSR 830256

Department of Civil Engineering and Engineering Mechanics
University of Arizona
Tucson, Arizona 85721

July 1988

FURTHER DEVELOPMENT OF HIERARCHICAL PLASTICITY MODEL FOR DISCONTINUITIES

INTRODUCTION

Discontinuities or joints play a significant role in the behavior of structures founded on discontinuous rocks (or soils) which contribute an important source of weakness. Although modern computational techniques make numerical analysis possible, proper constitutive relations for discontinuities must be used in order to obtain reliable and useful results.

Discontinuity, by definition, is the boundary region between two materials. Usually it is weaker compared to surrounding intact materials. The geometry and roughness of the joint walls play a significant role in the shear strength and deformation of the joint. The opposite walls of the discontinuity have only finite contacts, air, liquid and filling materials are often present, and it possesses negligible strength in tension. As a consequence, its behavior is quite different from that of continua surrounding it.

The above factors make modelling of a discontinuity difficult. Empirical models can usually only be applicable for limited situations. Here, a general model based on the plasticity theory is developed which can be applied to a large group of discontinuities or joints. Since the plasticity theory was originally developed for continuum, some modifications to the plasticity theory for joints are needed in order to accommodate the special features of discontinuities.

This model represents a generalized and modified development of the model presented in Part A of this report. The proposed generalized model allows for detailed consideration of the normal and shear components of the joint response, and is verified for both normal stress and normal stiffness controlled conditions.

STRESSES AND DISPLACEMENTS

Stresses

A joint is usually idealized as a planar thin layer and only the stresses on this plane are considered (Fig. 1). Thus, only the normal stress σ_n and the shear stress τ are included in the modeling (Fig. 2), and the influences of the other stress components are ignored. Since due to the roughness of the joint, there are only finite number of contacts and stresses are concentrated at these contacts, an assumption is also made that the stresses on the joint plane are uniformly distributed so the stresses are nominal rather than actual.

Normal Displacement

Under normal compression, the joint will be compressed, and under tension, the joint can separate. During shear, dilation may occur (Fig. 3), and this dilation is caused by the sliding of the asperities on the opposite walls of the joint. These compressions, separations and dilations are called normal displacements since they occur in the normal direction to the joint plane.

The total normal displacement v can be decomposed into three parts: the elastic and plastic deformation of the asperities v_e and v_p, respectively, and the slip displacement v_s, as

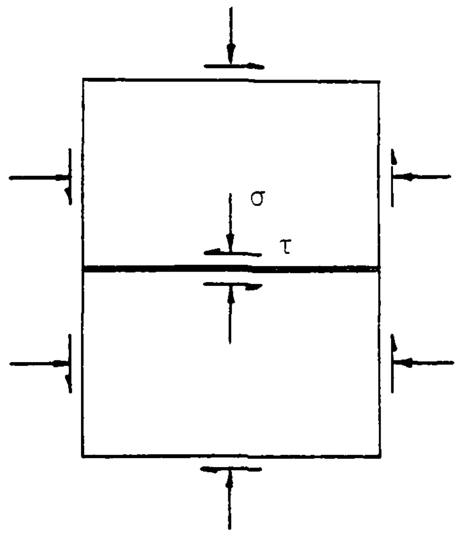

Fig. 1 Stresses On and Surrounding Joint or Discontinuity

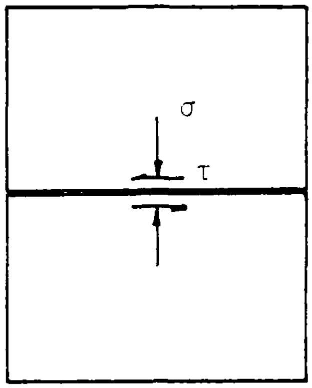

Fig. 2 Simplified Stress State on Joint or Discontinuity

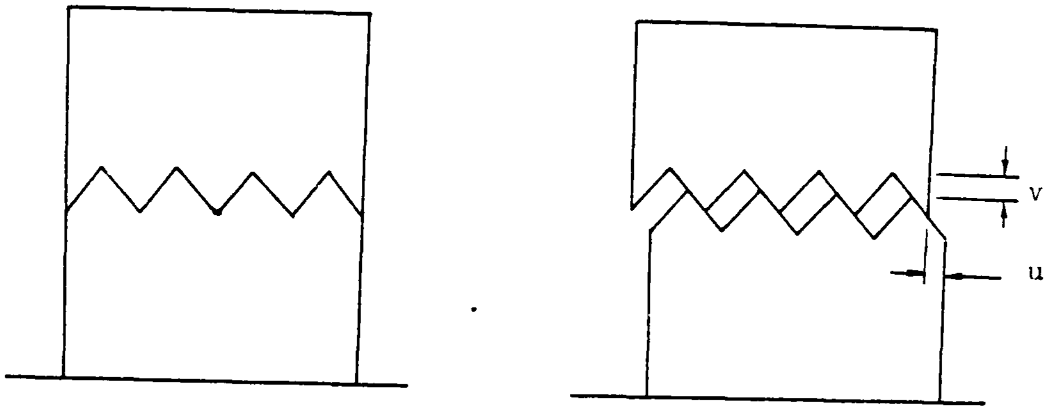

Fig. 3 Displacements for Joint or Discontinuity

$$v = v_e + v_p + v_s$$
$$= v_e + v_r \tag{1}$$

where v_r is called the relative normal displacement.

The normal slip displacement v_s is the projection of the sliding and slip displacement on the normal direction. In a normal compression test of an initially mated joint, the normal compression is mainly due to the deformation of the asperities and the normal slip displacement is negligible. On the other hand, however, in a shear test with constant normal stress, the vertical slip displacement is a major part of the dilation and the elastic and plastic deformations of the asperities are relatively small compared to the slip displacement.

Tangential Displacement

Similar to the normal displacement, the tangential displacement u can be decomposed into three parts:

$$u = u_e + u_p + u_s \tag{2}$$

where u_e, u_p represent the elastic and plastic shearing deformations of the asperities, respectively, and u_s is the tangential slip displacement.

It is very difficult to separate the plastic deformation u_p and the slip displacement u_s, so they can be combined as relative tangential displacement u_r, as

$$u_r = u_p + u_s \tag{3}$$

Usually, the plastic deformation u_p is quite small compared to the slip displacement u_s. In some cases, the plastic deformation of the asperities u_p is ignored in modeling.

Plasticity for Joints

The plasticity theory was developed for continua. In the plasticity theory, there is a yield function, F, that serves as a field equation, and a flow rule which determines the plastic deformation directions.

For joints, however, plasticity can not be directly used. Firstly, the joint is two-dimensional; hence, a special form of the yield function is needed. Secondly, the irrecoverable displacements are not necessarily plastic deformations, but a combination of sliding, slipping and plastic deformations of the asperities. Thus, the displacement has a preferred direction which is determined by the surface geometry of the joint. The plastic deformations of the asperities enter as a "correction" to this preferred direction. When the joint is relatively hard or the stress level is relatively low such that plastic deformations are very small compared to the slip displacement, we can neglect such "correction."

By specializing the general three-dimensional yield function for continua developed by Desai et al. [1], a yield function F for joints is obtained by Desai and Fishman [2], Fishman and Desai [3] and Fishman [4] as

$$F = \tau^2 + \alpha \sigma_n^{\ n} - \gamma \sigma_n^{\ 2} \qquad (4)$$

where n, γ are response functions and α is the hardening or growth function. The hardening function α can be expressed as

$$\alpha = \alpha_n \alpha_\tau \tag{5}$$

where α_n, α_τ are the normal and shear hardening functions, respectively.

The flow rule for joint is different from that of continua due to the existence of the slip displacement. The flow rule for the plastic deformation for the two-dimensional case can be expressed as

$$\begin{aligned} dv_p &= \lambda \frac{\partial Q}{\partial \sigma_n} \\ du_p &= \lambda \frac{\partial Q}{\partial \tau} \end{aligned} \tag{6}$$

where d denotes increment, Q is the plastic potential and λ is the scalar proportionality parameter.

For the joint, the plastic deformation is the difference between the relative displacement and the slip displacement, Eq. (1). Thus,

$$\begin{aligned} dv_r &= dv_s + dv_p = dv_s + \lambda \frac{\partial Q}{\partial \sigma_n} \\ du_r &= du_s + du_p = du_s + \lambda \frac{\partial Q}{\partial \tau} \end{aligned} \tag{7}$$

where dv_r, du_r are the incremental normal and the tangential relative displacements, respectively, and dv_s, du_s are the incremental normal and tangential slip displacements, respectively.

If the plastic deformation is negligible, then the relative displacement is approximately equal to the slip displacement.

Stress-Displacement Relations

Normal Compression Test

In the normal compression test, for initially mated joints, the shear stress τ and the tangential displacement are equal to zero. Thus, the yield function F reduces to F_n:

$$F_n = \alpha_n \sigma_n^{n} - \gamma \sigma_n^{2} = 0 \tag{8}$$

and from the consistency condition, dF = 0;

$$dF_n = \frac{\partial F_n}{\partial \sigma_n} d\sigma_n + \frac{\partial F_n}{\partial v_p} dv_p = 0 \tag{9}$$

there, the normal displacement is mainly deformation of the asperities and the slip displacement is assumed to be zero. So,

$$dv = dv_e + dv_p \tag{10}$$

The elastic vertical displacement dv_e is given by

$$dv_e = \frac{d\sigma_n}{K_n} \tag{11}$$

where K_n is the unloading elastic normal stiffness of the joint. The plastic normal displacement is obtained from Eq. (9) as

$$dv_p = - \left[\left(\frac{\partial F_n}{\partial \sigma_n}\right) / \left(\frac{\partial F_n}{\partial v_p}\right) \right] d\sigma_n \tag{12}$$

So the total displacement is

$$dv = [(\frac{1}{K_n} - (\frac{\partial F_n}{\partial \sigma_n})/(\frac{\partial F_n}{\partial v_p})]\ d\sigma_n \tag{13}$$

Different models can be obtained by choosing different hardening function α_n. One possible model, based on observed laboratory behavior, can be expressed as

$$\alpha_n = \gamma \text{ Exp } [(2-n)\ (h_1\ v_p + h_2)] \tag{14}$$

where γ, n, h_1 and h_2 are parameters and v_p is the normal plastic compression.

Shear Tests

In the previous section, the normal compression behavior (or the incremental normal stress and normal compression) is modeled by using the yield function F_n. During shear, if we assume that the normal stress and normal plastic deformation relation is not changed, then we can assume that

$$F_n = \alpha_n\ \sigma_n^{\ n} - r\ \sigma_n^{\ 2} = 0 \tag{15}$$

still holds. The yield function relevant to the shear condition can be expressed as, Eq. (4),

$$F_\tau \quad \tau^2 + \alpha_\tau\ \alpha_n\ \sigma_n^{\ n} - \gamma\ \sigma_n^{\ 2} \tag{16}$$

By applying Eq. (15), we obtain

$$F_\tau = \tau^2 + \gamma\ (\alpha_\tau - 1)\ \sigma_n^{\ 2} = 0 \tag{17}$$

where α_τ is the hardening or growth function and can be expressed as

$$\alpha_\tau = 1 - \frac{1}{\gamma} \text{ Exp } (h_3) \times u_r^{\ h_4} \tag{18}$$

and

$$h_3 = u\,\sigma_n + b$$
$$h_4 = c\,\sigma_n + d \qquad (19)$$

where a, b, c, d and γ are constants, and u_r is the relative tangential displacement.

The total incremental displacements can now be expressed as

$$du = du_r + du_e = du_r + \frac{d\tau}{K_\tau} \qquad (20)$$

$$dv = dv_r + dv_e = dv_r + \frac{d\sigma_n}{K_n} \qquad (21)$$

where K_τ, K_n are the unloading shear and normal stiffnesses, respectively.

Because the plastic deformations of the asperities are in many cases very small compared to the slip displacement. The flow rule can be approximately assumed as

$$dv_r = g\,du_r \qquad (22)$$

where g is the geometrical function, as

$$g = \frac{df}{du_r} \qquad (23)$$

where

$$f = v_u\,[1 - \mathrm{Exp}\,(-k\,u_r^{\,m})] \qquad (24)$$

where v_u is the ultimate dilation and k, m are constants.

Shear tests can be classified as (a) constant normal force stress and (b) constant normal stiffness. However, very often both situations may be coupled.

For a shear test with a constant normal loading stiffness, category (b), k_d, (or a CNK test), when there is dilation dv, the loading system will increase the normal stress such that

$$d\sigma_n = K_d \, dv \tag{25}$$

This test is denoted by CNK here. Shear test with constant normal stress, category (a), as denoted by CNS.

From the consistency condition:

$$dF_\tau = \frac{\partial F_\tau}{\partial \tau} d\tau + \frac{\partial F_\tau}{\partial \sigma_n} d\sigma_n + \frac{\partial F_\tau}{\partial u_r} du_r + \frac{\partial F_\tau}{\partial v_r} dv_r = 0 \tag{26}$$

and by using Eqs. (20) and (26), we obtain

$$d\tau = \frac{1}{\frac{1}{K_\tau} + \frac{1}{D}} du \tag{27}$$

where

$$D = \frac{-\frac{\partial F_\tau}{\partial \tau}}{\frac{K_n K_d g}{K_n - K_d} \frac{\partial F_\tau}{\partial \sigma_n} + \frac{\partial F_\tau}{\partial u_r} + g \frac{\partial F_\tau}{\partial u_r}} \tag{28}$$

and

$$dv = \frac{K_n g}{K_n - K_d} \left(du - \frac{d\tau}{K_\tau}\right) \tag{29}$$

and

$$d\sigma_n = \frac{K_d \; g}{K_n - K_d} \left(du - \frac{d\tau}{K_\tau}\right) \tag{30}$$

PARAMETERS

The parameters to be determined are the elastic stiffness, the constants in the hardening functions, the constants in the geometrical function and the constant for the ultimate state.

Elastic Constants

The normal compression elastic stiffness K_n and the shear elastic stiffness K_τ are the elastic constants used in the model. They are assumed equal to the unloading stiffness in the normal compression test (Fig. 4) and the shear test (Fig. 5), respectively.

Hardening Functions

The constants in the normal hardening function α_n are h_1 and h_2. They are obtained from the $\ell n\ \sigma_n \sim v_p$ plot (Fig. 6), and

$$\ell n\ \sigma_n = h_1\, v_p + h_2 \tag{31}$$

The parameters in the shear hardening function α_τ are h_3 and h_4. They are obtained from the $\ell n\ (\frac{\tau}{\sigma_n}) \sim \ell n\ u_r$ plot (Fig. 7) and

$$\ell n\ \left(\frac{\tau}{\sigma_n}\right) = h_3\ \ell n\ u_r + h_4 \tag{32}$$

and h_3, h_4 are further determined by

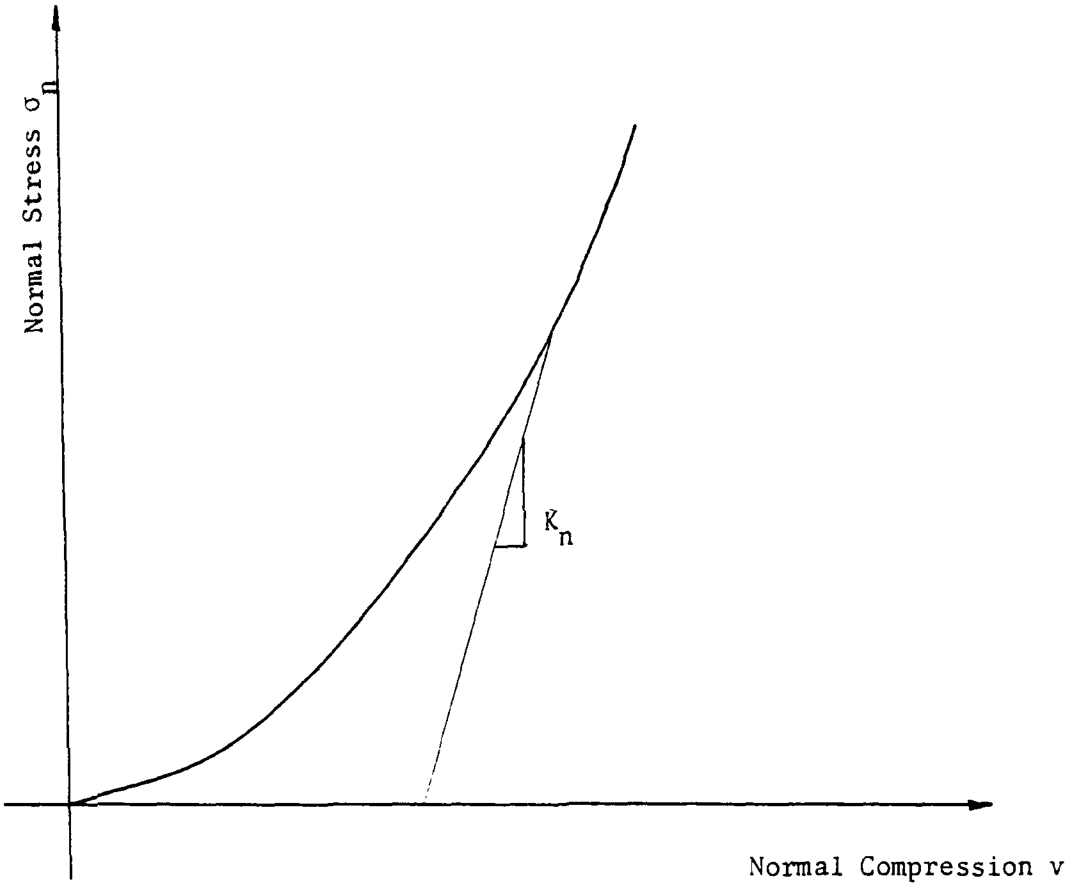

Fig. 4 Unloading Elastic Stiffness K_n from Normal Compression Test

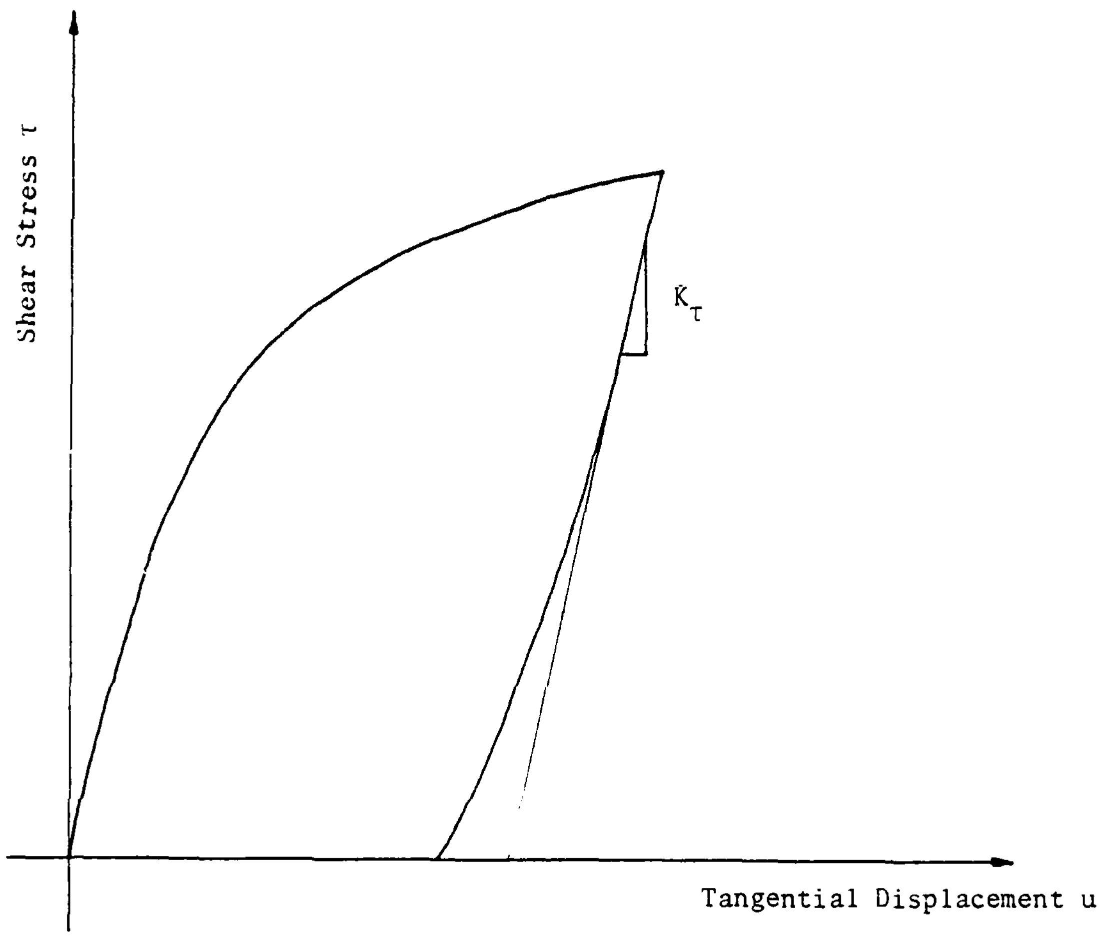

Fig. 5 Unloading Elastic Stiffness K_τ from Shear Test

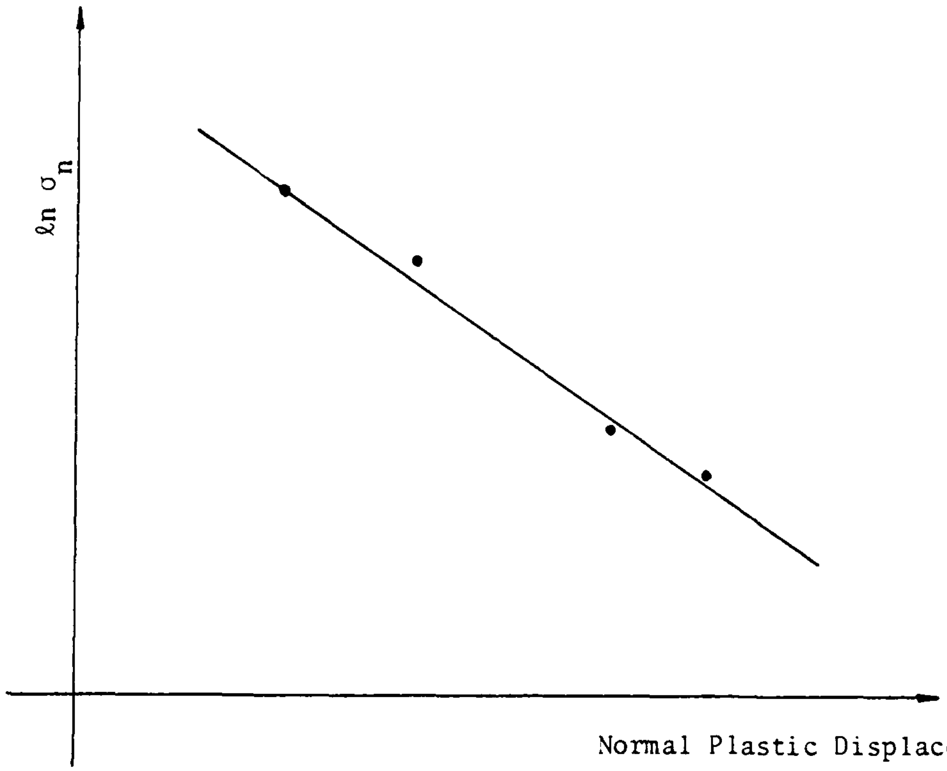

Fig. 6 Plot to Find Constants h_1 and h_2

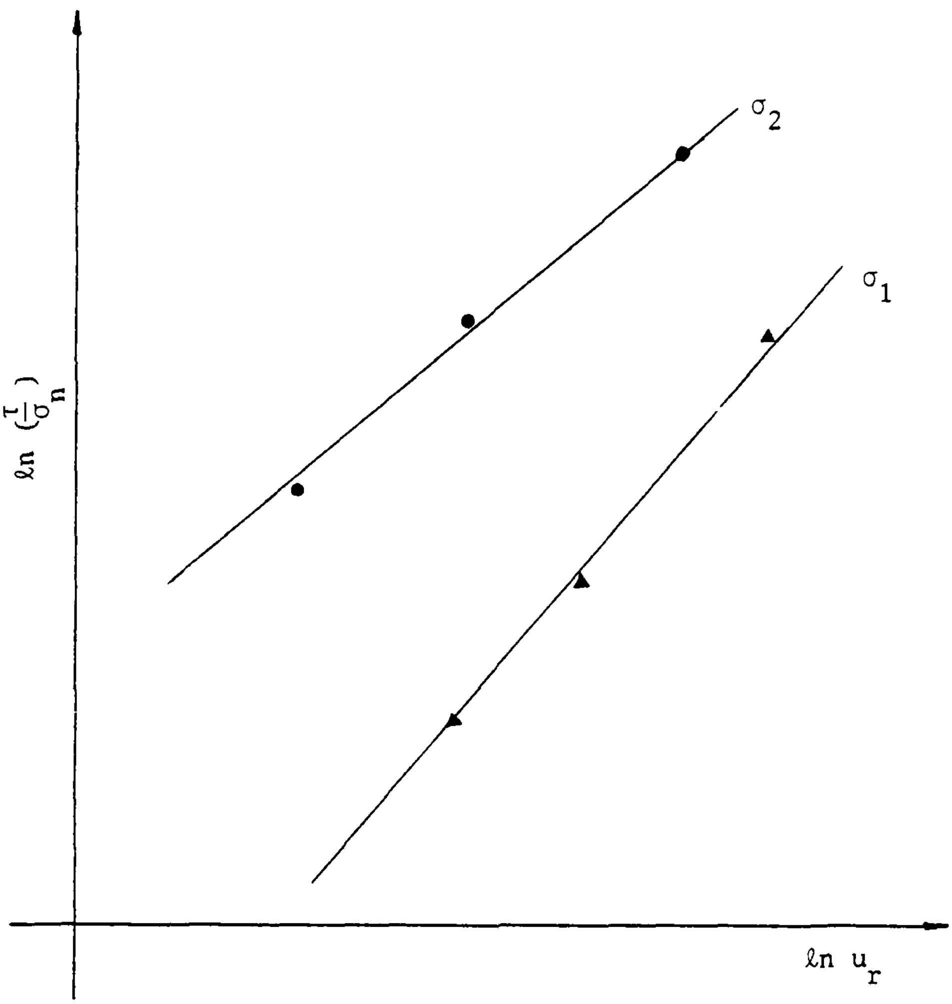

Fig. 7 Plot to Find Parameters h_3 and h_4 Which Leads to Find Constants a, b, c, d

$$h_3 = a\,\sigma_n + b$$
$$h_4 = c\,\sigma_n + d \tag{33}$$

where a, b, c, d are constants.

Geometrical Function

To determine the geometrical function, three constants are needed: v_u, k and m, and the geometrical function g is expressed as

$$g = \frac{df}{du_r} \tag{34}$$

where

$$f = v_u\,[1 - \mathrm{Exp}\,(-k\,u_r^{\,m})] \tag{35}$$

These constants are obtained from the $\ell n\,[\ell n\,(\frac{v_u}{v_u - v_r})] - \ell n\,u_r$ plot (Fig. 8), where v_r is the relative normal displacement.

Constant of Ultimate State

At the ultimate state, $\alpha = 0$, so the yield function becomes

$$F_f = \tau_f^{\,2} - \gamma\,\sigma_n^{\,2} = 0 \tag{36}$$

so

$$\gamma = \frac{\tau_f^{\,2}}{\sigma_n^{\,2}} \tag{37}$$

and γ is obtained by best curve fitting to the Df ultimate stress states for different normal stresses (Fig. 9a).

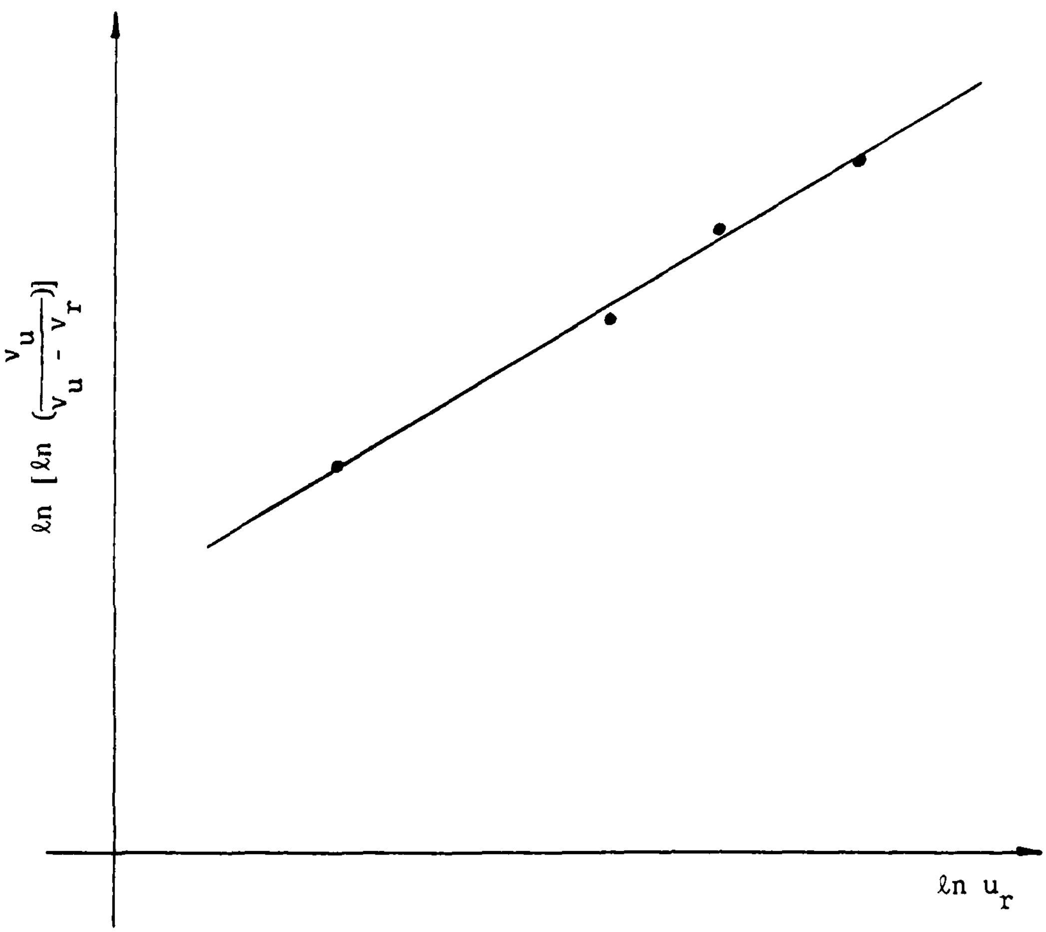

Fig. 8 Plot to Find Constants k and m

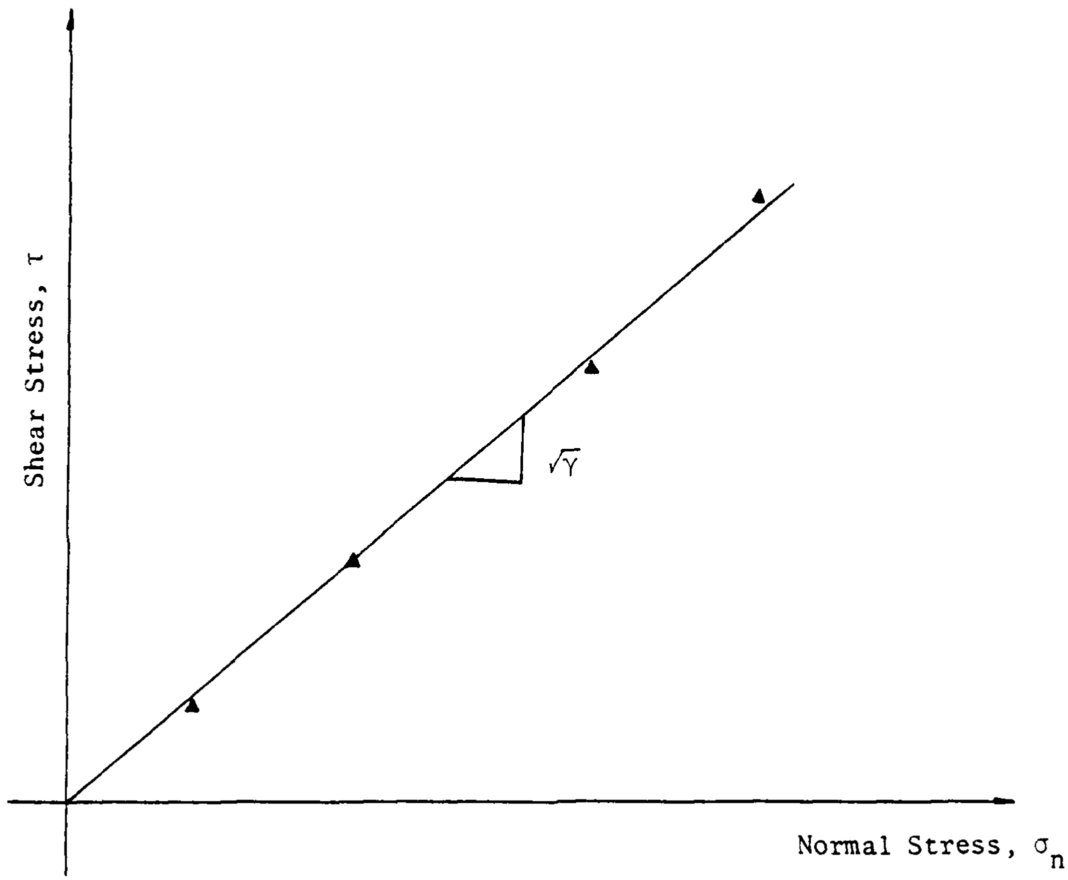

Fig. 9a Plot to Find Constant γ

Constant n

The constant n is usually determined at the phase change point, Fig. 9(b), and for a joint, this point is often hard to select. As this time, based on previous tests [4], n = 2.5 is adopted. However, later it would be further investigated.

BACK PREDICTIONS AND VERIFICATION

In order to verify the model, three sets of test data are used. The first data set is from a paper by Yoshinaka and Yamabe [5]. The joint is artificially made by cutting an originally intact rock with the diamond saw. The tests performed are the normal compression test and the shear test with constant normal stresses.

The second data set is from the tests performed by Fishman [4]. The joint is prepared by casting the concrete against a saw-shaped metal sheet. Here, only the shear tests with constant normal stresses are used in back predictions.

The third data set is from Dight and Chiu [6]. The joints are very rough surfaces of rock on concrete. These shear tests involve constant normal stiffness.

The above three tests are designated as normal compression test (NCT), the shear test with constant normal stresses (CNS), and the shear test with constant normal stiffness (CNK).

For back predicting the NCT tests, the constants required are K_n, h_1 and h_2. For CNS and CNK tests, the constants required are K_τ, γ, a, b, c, d and v_u, k, m.

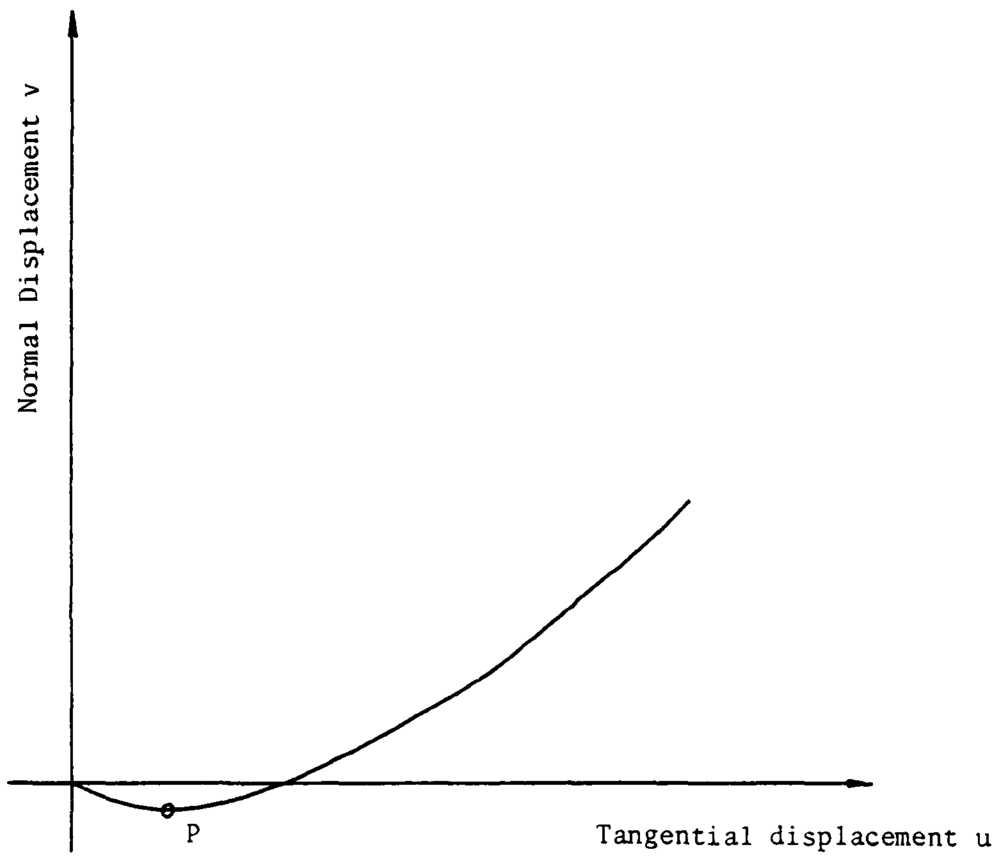

Fig. 9b Phase Change Point P

Table 1 gives the constants for all three data sets. Table 2 indicates all the figures (from Fig. 10 to Fig. 17) that show the correlation between the data and the constants found. For all the data sets, the correlation is satisfactory.

Back predictions are made and compared to the data for all three data sets (Fig. 18 - Fig. 23).

Figures 18 and 19 back predict the relations of normal stress σ_n and normal displacement v and the relation of shear stress τ and tangential displacement u for data set #1. The results are very satisfactory.

Figures 20 and 21 give the back prediction of the relation of shear stress τ and tangential displacement u and the relation of dilation v and tangential displacement u for data set #2. This is a shear test with constant normal stresses. But under different normal stresses, the dilations are almost the same since the joint is very stiff and the dilations are mostly rigid slips.

Figures 22 and 23 give the back predictions for data set #3. Because this is a shear test with constant normal stiffness CNK, the stress-displacement relations are quite different from the CNS tests. A very interesting point is that while the normal stiffness keeps constant, the normal stress σ increases and the shear stress τ decreases after it reaches a peak. This is a complicated situation since the softening occurs in shear but strengthening or hardening happens in normal direction. The model catches this feature satisfactorily.

Summary and Conclusions

A plasticity model for joints and discontinuities is presented. Due to many special features of the joints and discontinuities, the plasticity

Table 2. Figures in Finding Constants

Test Type / Data Set	NCT	CNS	CNK
Set #1 From Yoshinaka and Yamabe (5)	Fig. 10	Fig. 11 Fig. 12	
Set #2 From Fishman (4)		Fig. 13 Fig. 14 Fig. 15	
Set #3 From Dight and Chin (6)			Fig. 16 Fig. 17

Table 1. Constants for Various Tests

1 psi = 6.89 Kpa
1 in = 2.54 cm

Data Set/ Test Type	NCT Test	CNS Test	CNK Test
Set #1 From Ref. 1	K_n = 17.22 Mpa/mm	K_τ = 12.71 Mpa/mm	
	h_1 = -9.6761	γ = 0.4447	
	h_2 = 2.9135	a = 0.00252	
		b = 0.21329	
		c = 0.04426	
		d = - 0.4444	
Set #2 From Ref. 4		K_τ = 40,000 psi/in	
		γ = 0.5146	
		a = 0.0014	
		b = 0.0331	
		c = 0.00005	
		d = - 0.54217	
		v_u = 0.008 (in)	
		k = 0.7604	
		m = 1.9888	
Set #3 From Ref. 6			K_τ = 1000 Kpa/mm
			γ = 1.9888
			a = - 0.00275
			b = - 0.31043
			c = 0.00936
			d = 0.20646
			v_u = 2.11 (mm)
			k = 0.2639
			m = 0.7848

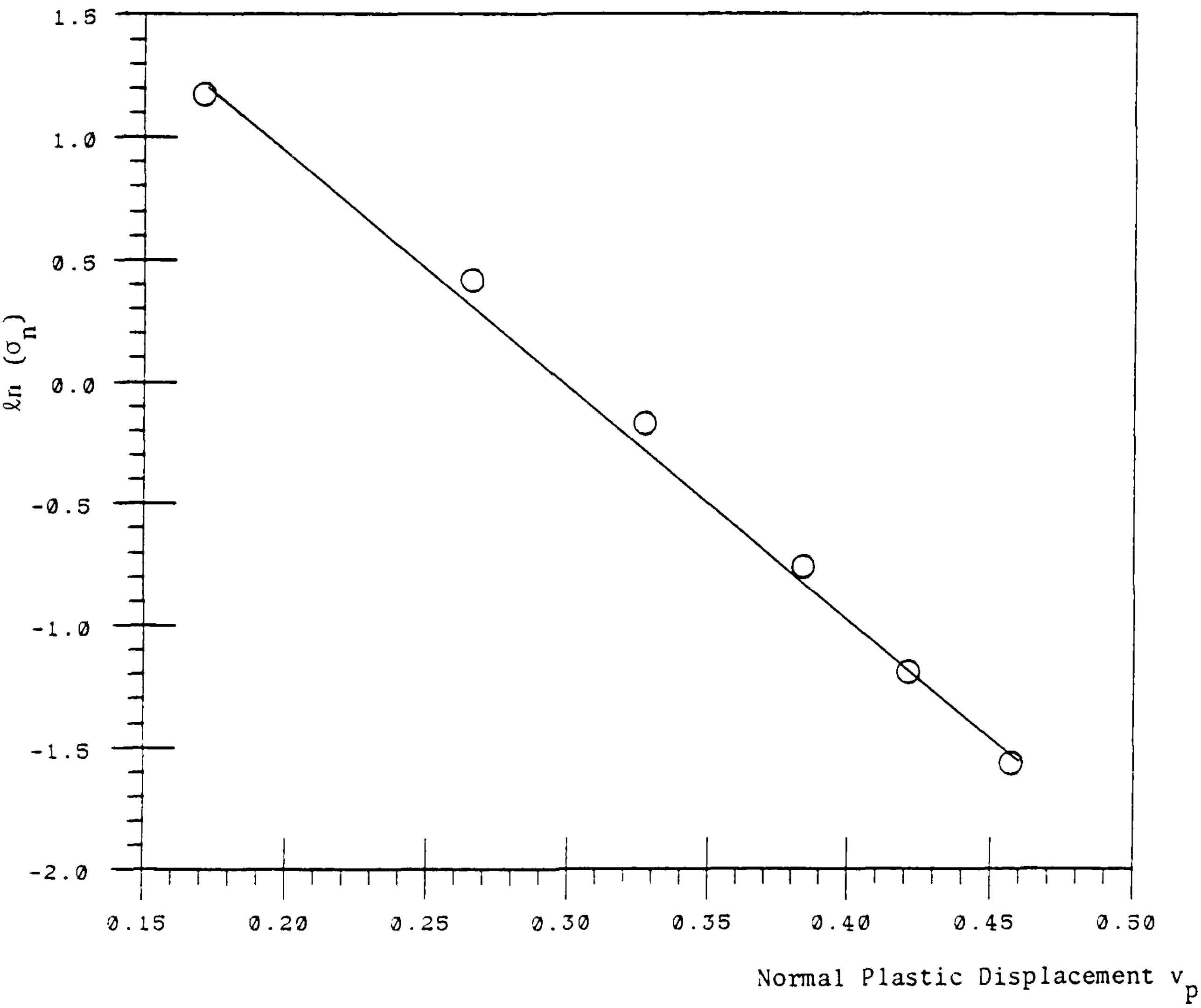

Fig. 10 Plot to Find Constants h_1 and h_2
(Data Set #1)

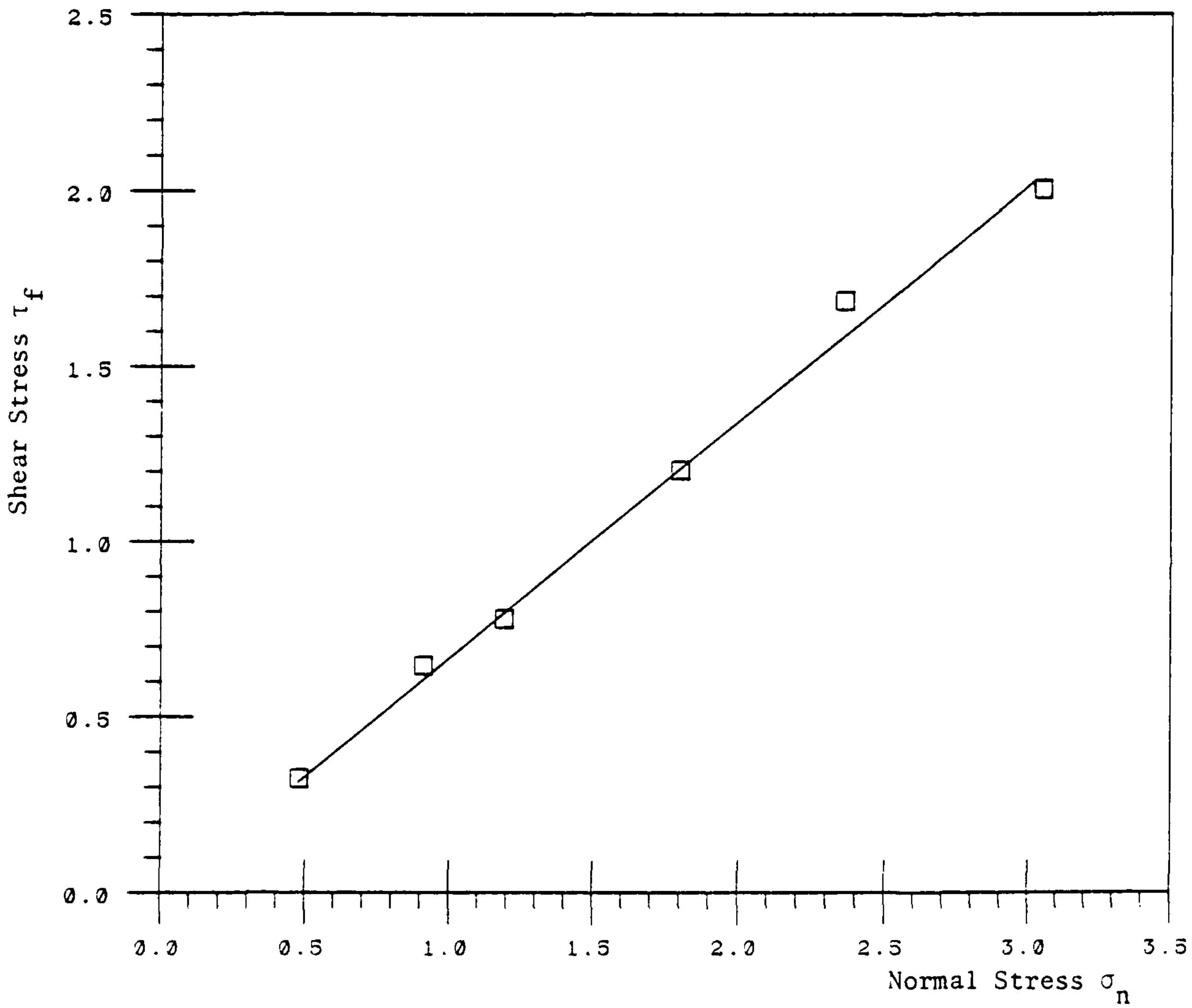

Fig. 11 Plot to Find Constant γ (Data Set #1)

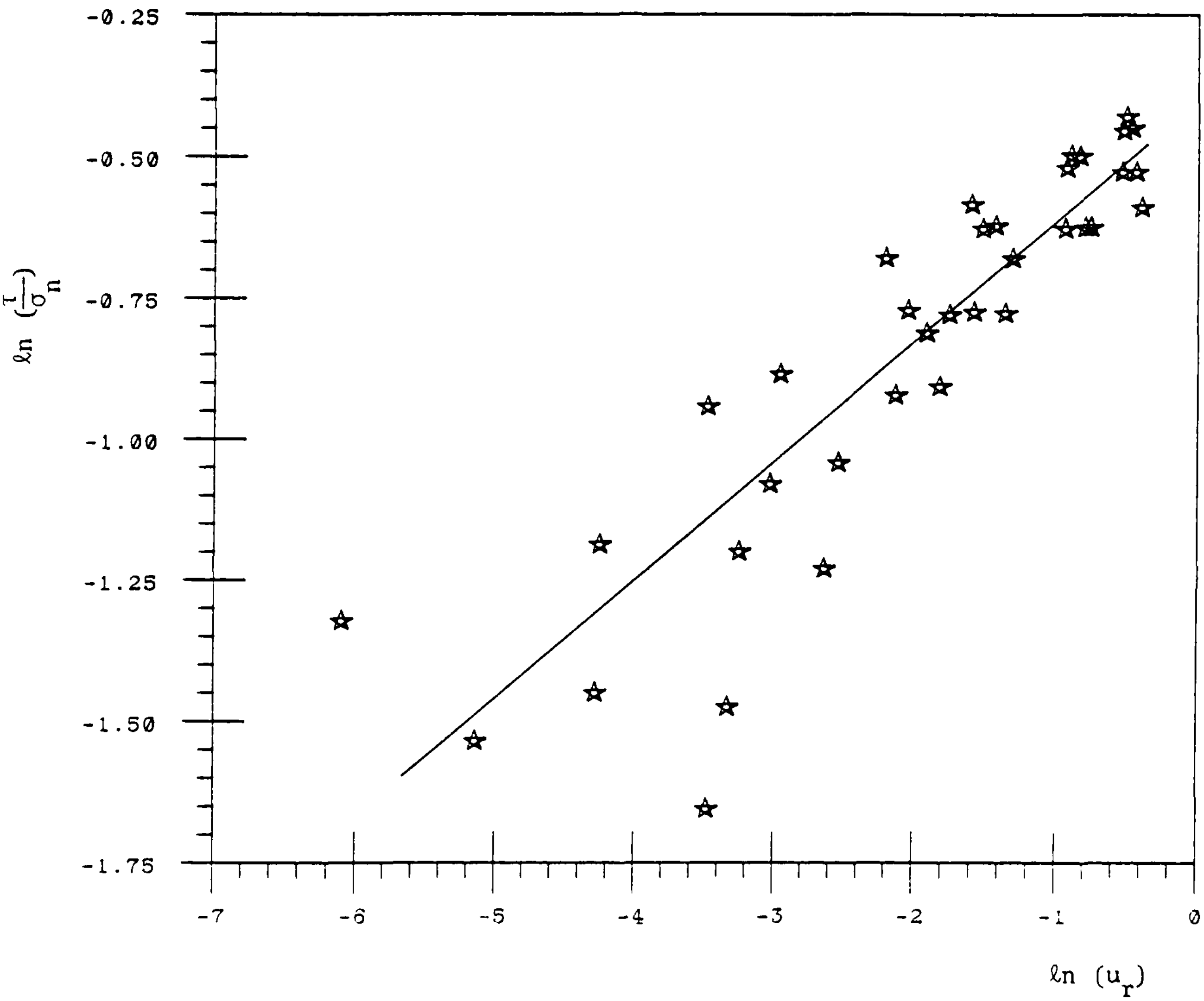

Fig. 12 Plot to Find h_3 and h_4 (Data Set #1)

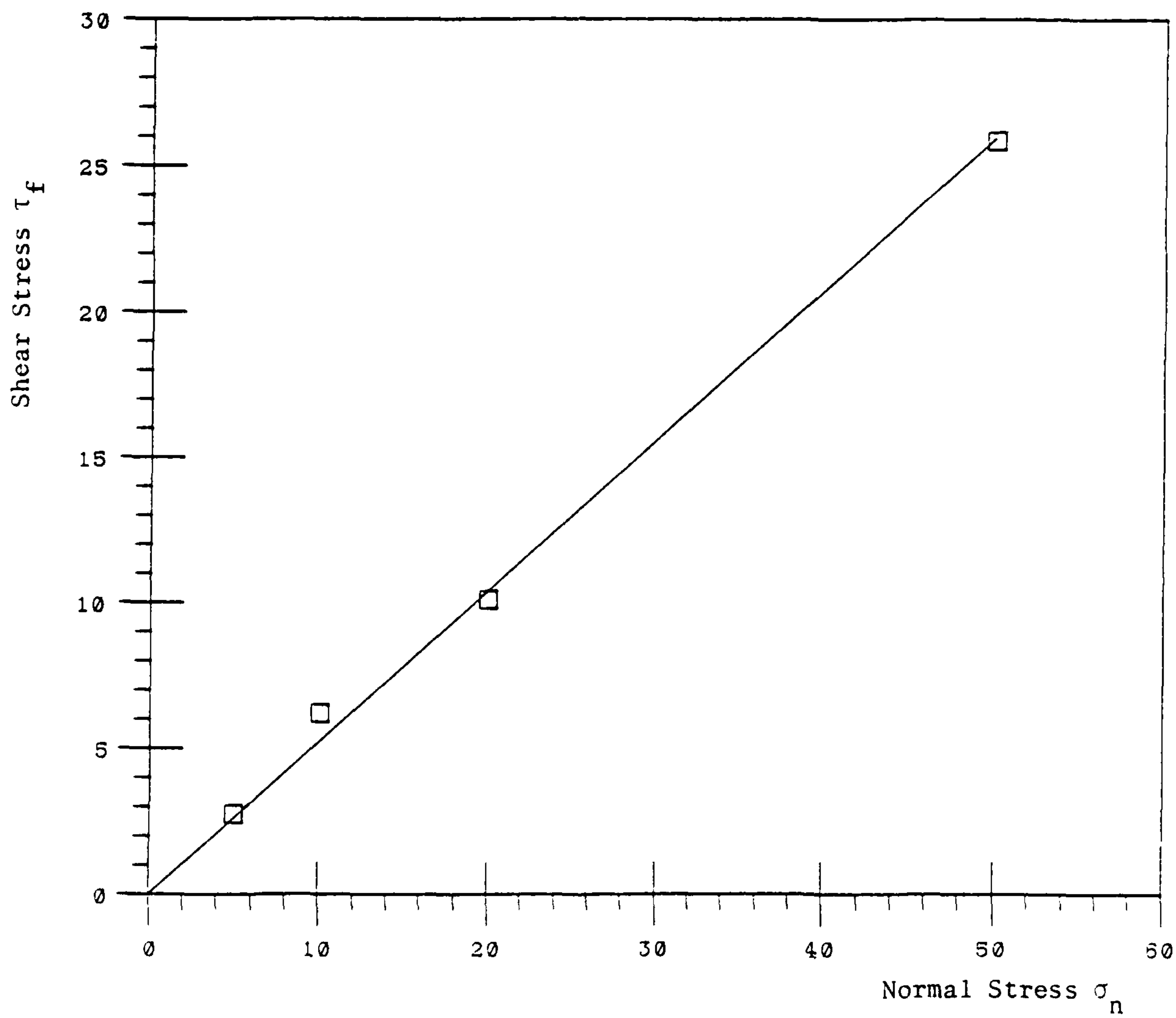

Fig. 13 Plot to Find Constant γ (Data Set #2)

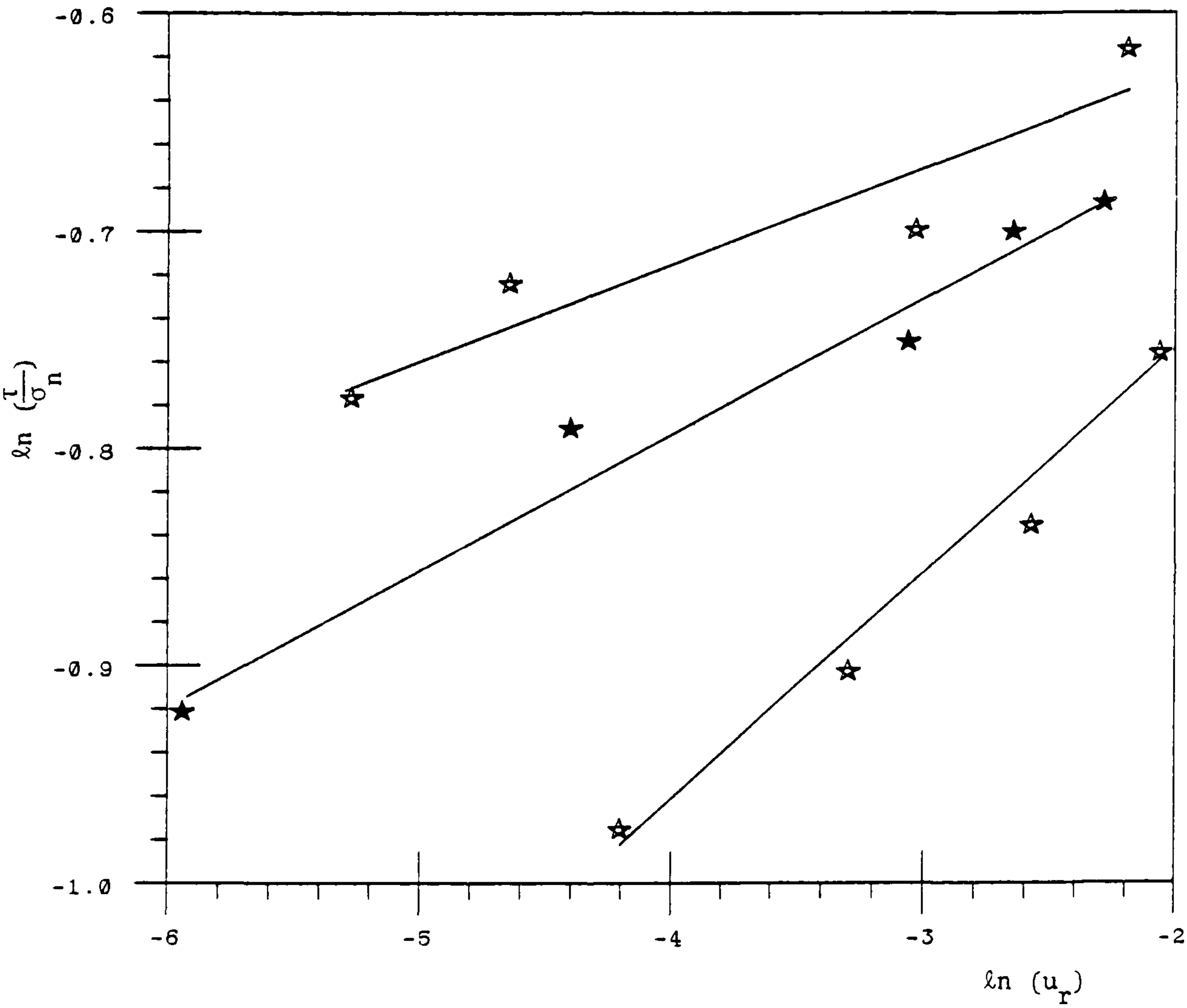

Fig. 14 Plot to Find h_3 and h_4 (Data Set #2)

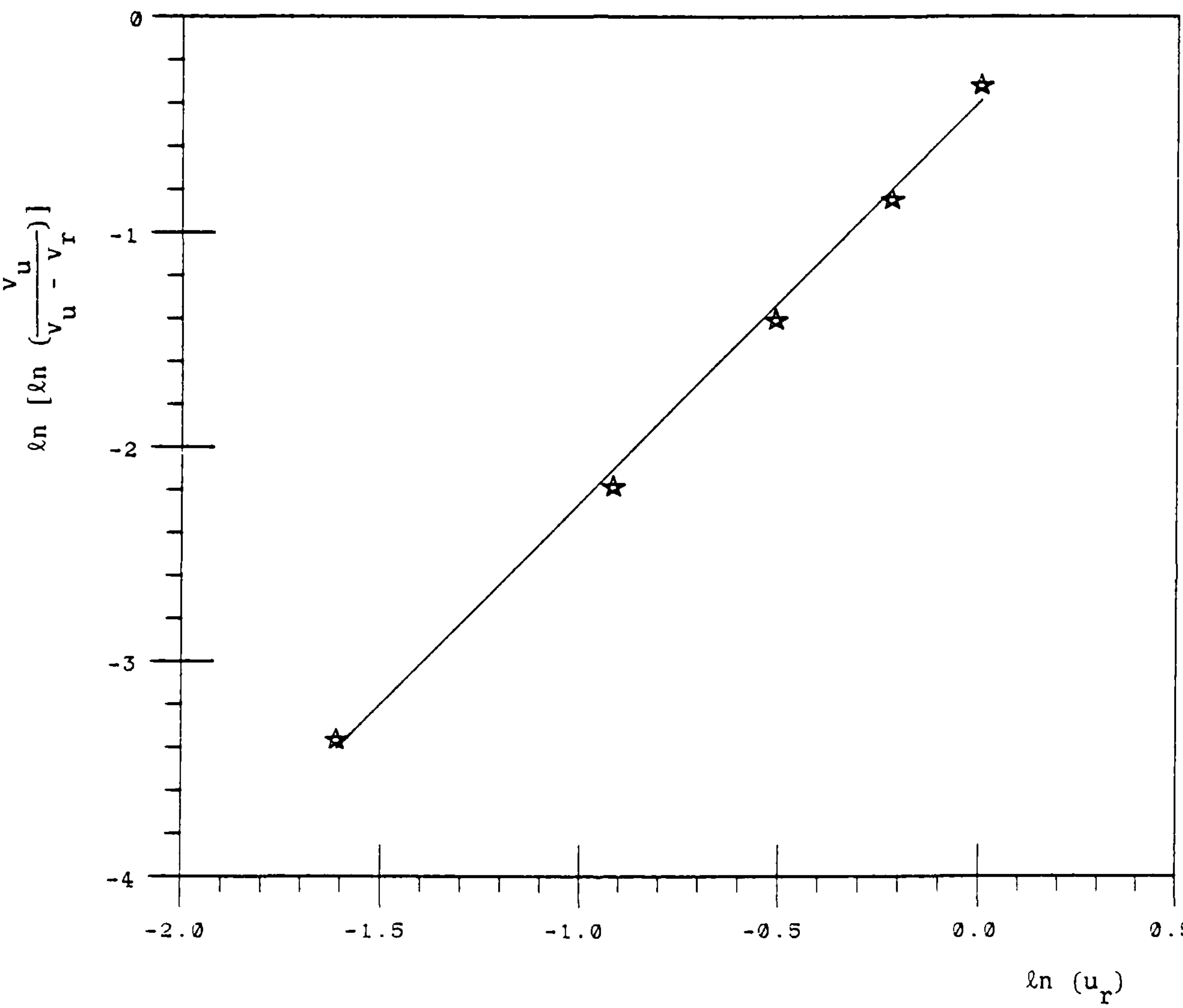

Fig. 15 Plot to Find Constants k and m (Data Set #2)

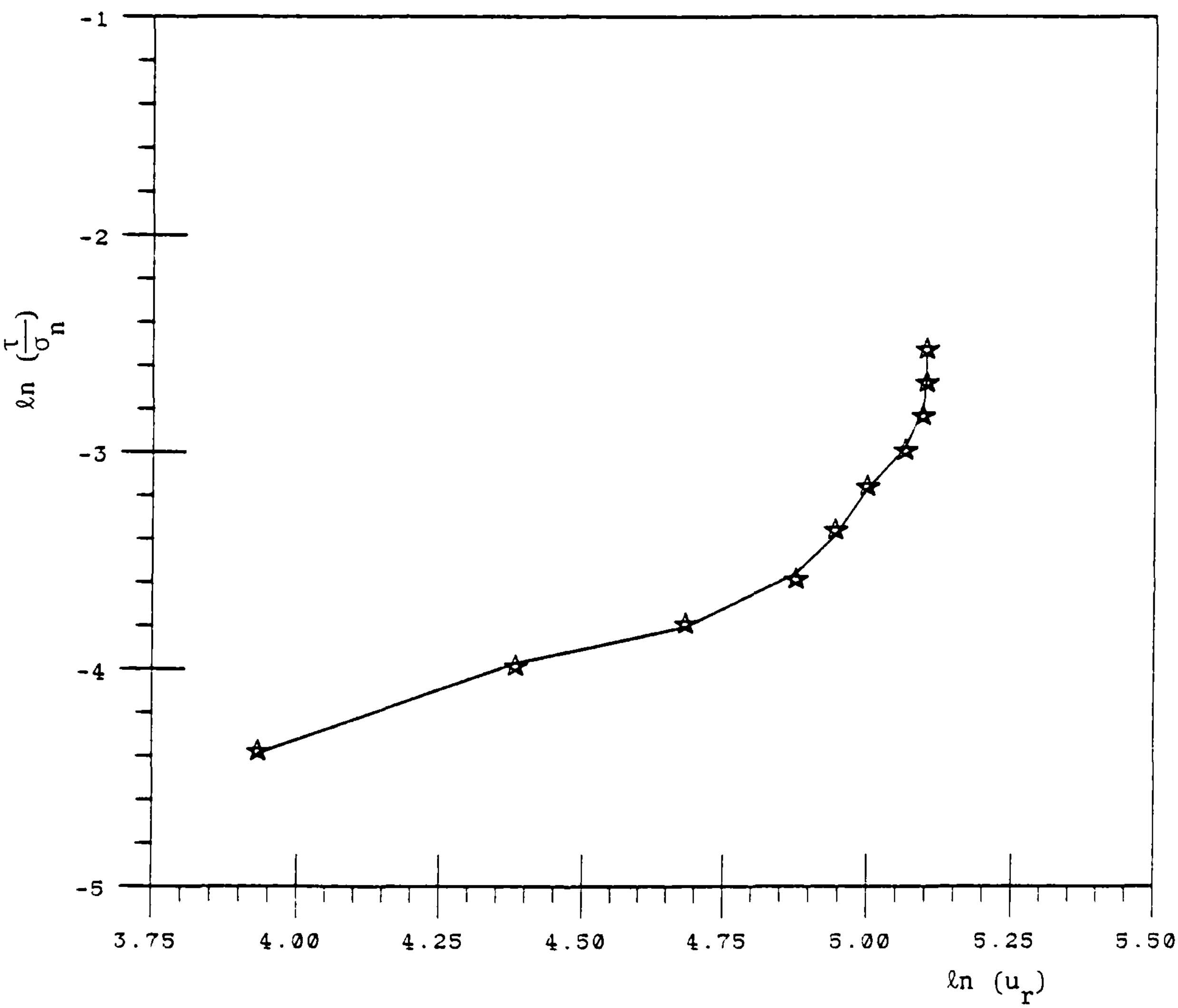

Fig. 16 Plot to Find h_3 and h_4 (Data Set #3)

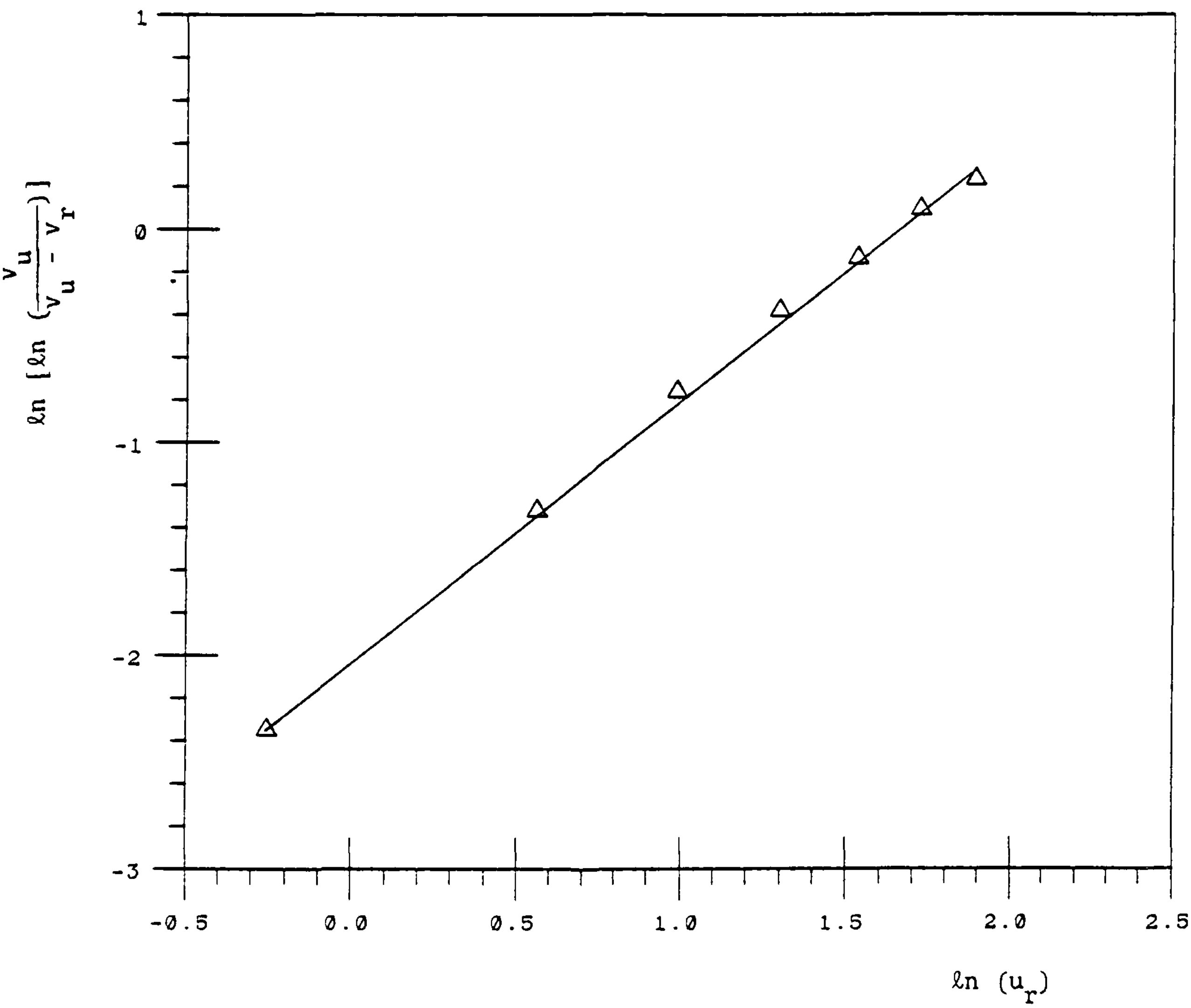

Fig. 17 Plot to Find Constants h_3 and h_4 (Data Set #3)

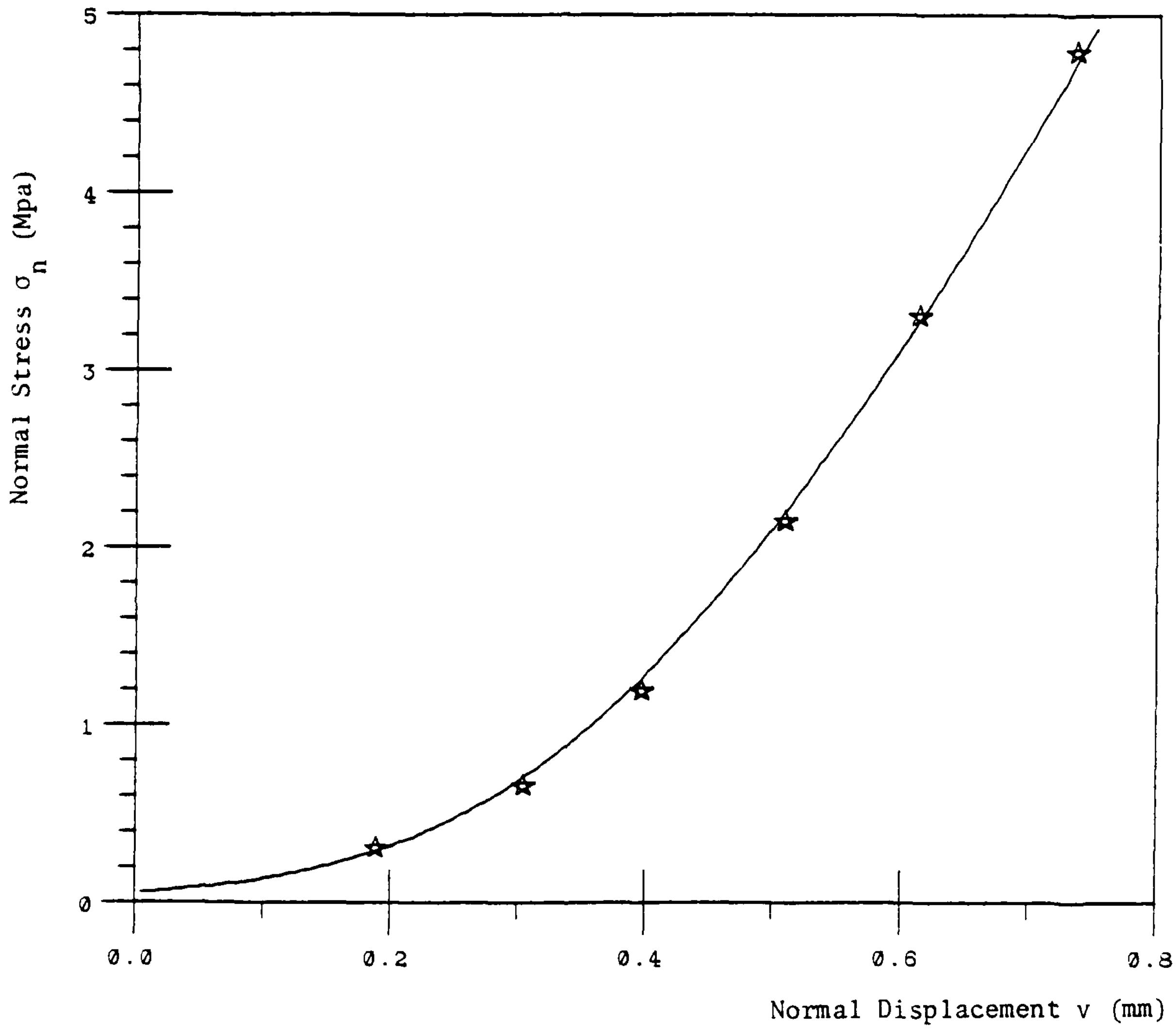

Fig. 18 Back Prediction - Normal Stress σ_n and Normal Displacement v Relation in Normal Compression Test (Data Set #1)

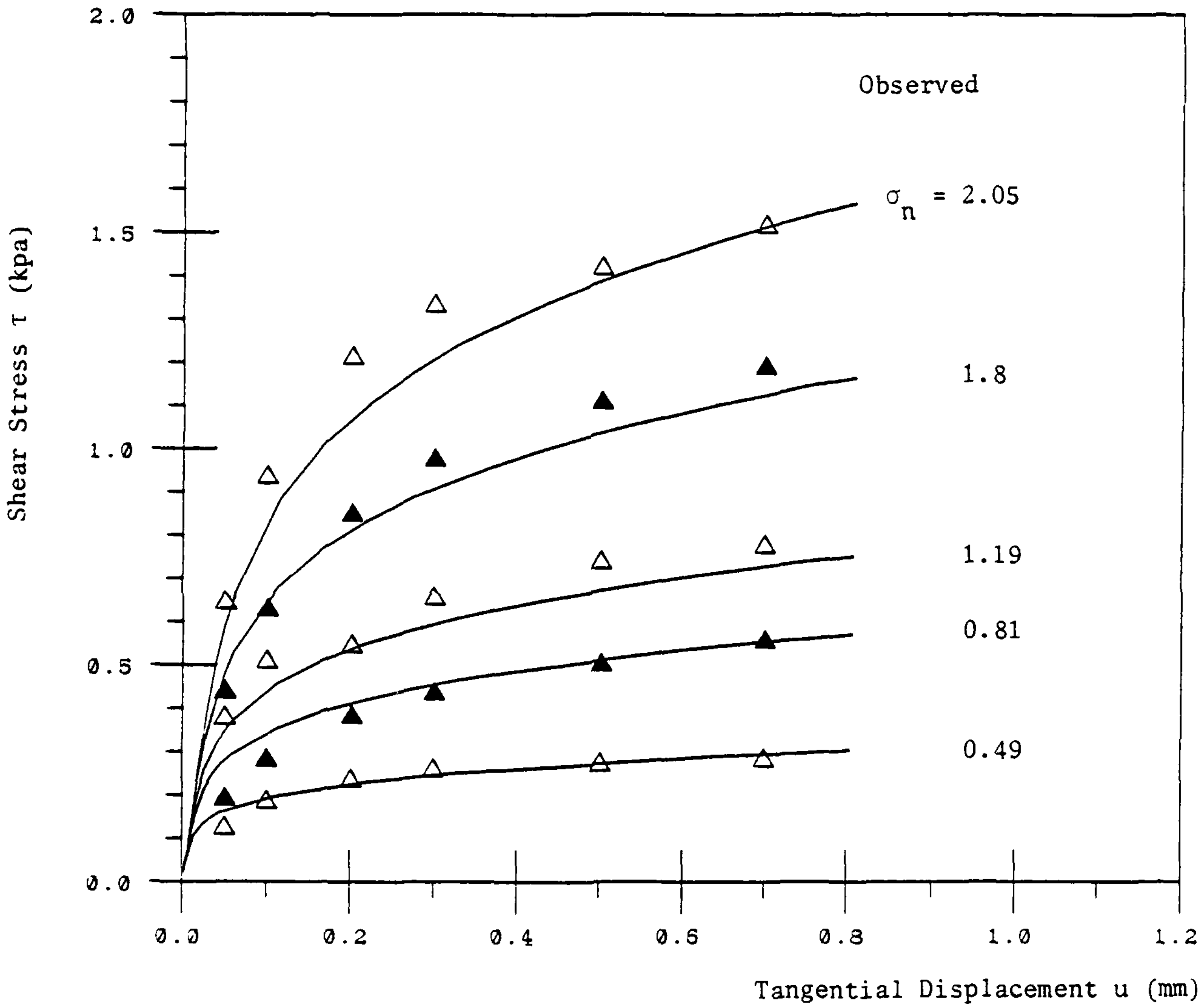

Fig. 19 Comparisons Between Back Predictions and Observations for Shear Stress and Shear Displacement Relations Under Various Normal Stresses σ_n in Shear Tests (CNS) (Data Set #1)

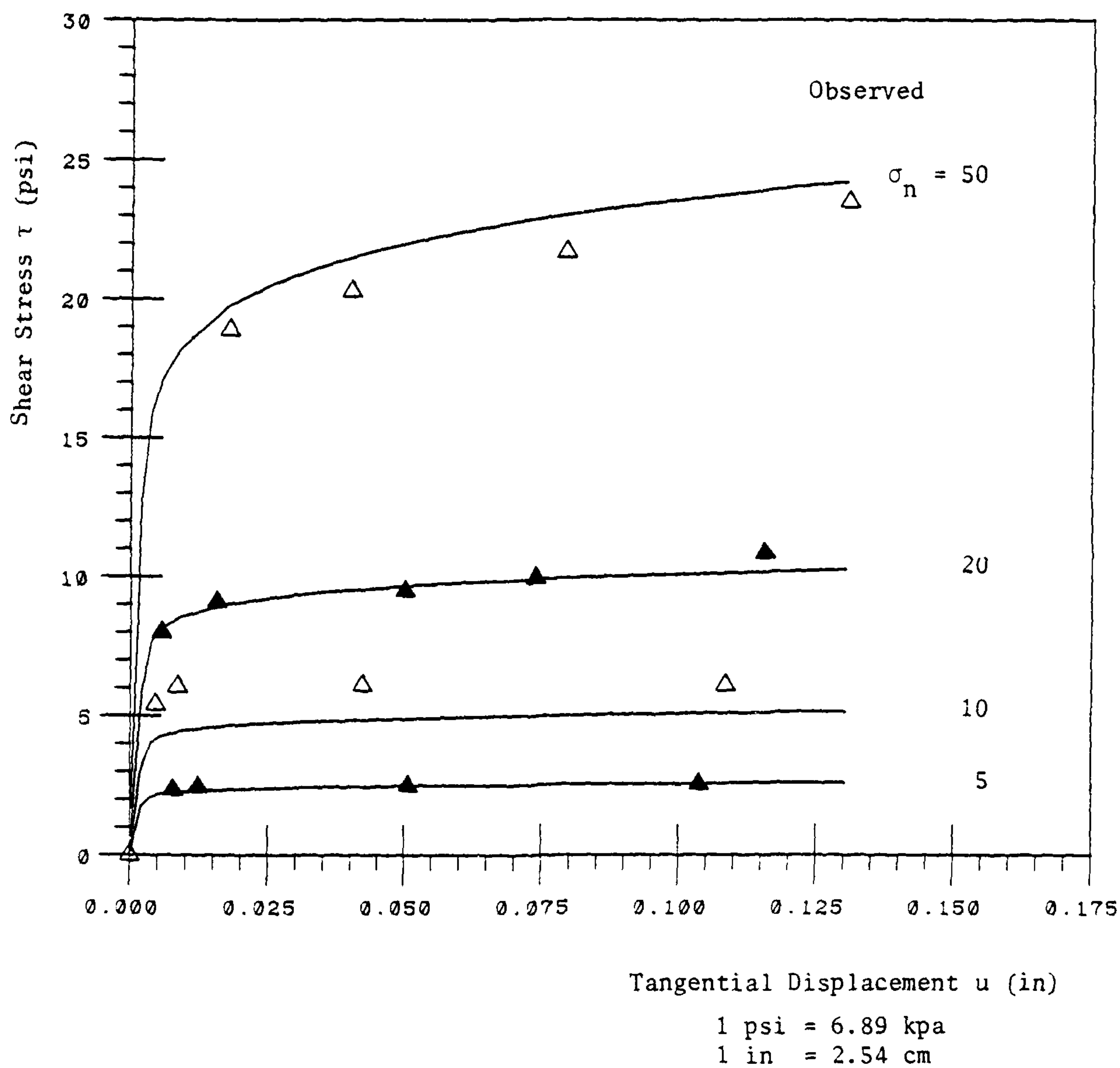

Fig. 20 Comparisons Between Back Predictions and Observations for Shear Stress and Tangential Displacement Relation Under Various Normal Stresses σ_n in Shear Test (CNS) (Data Set #2)

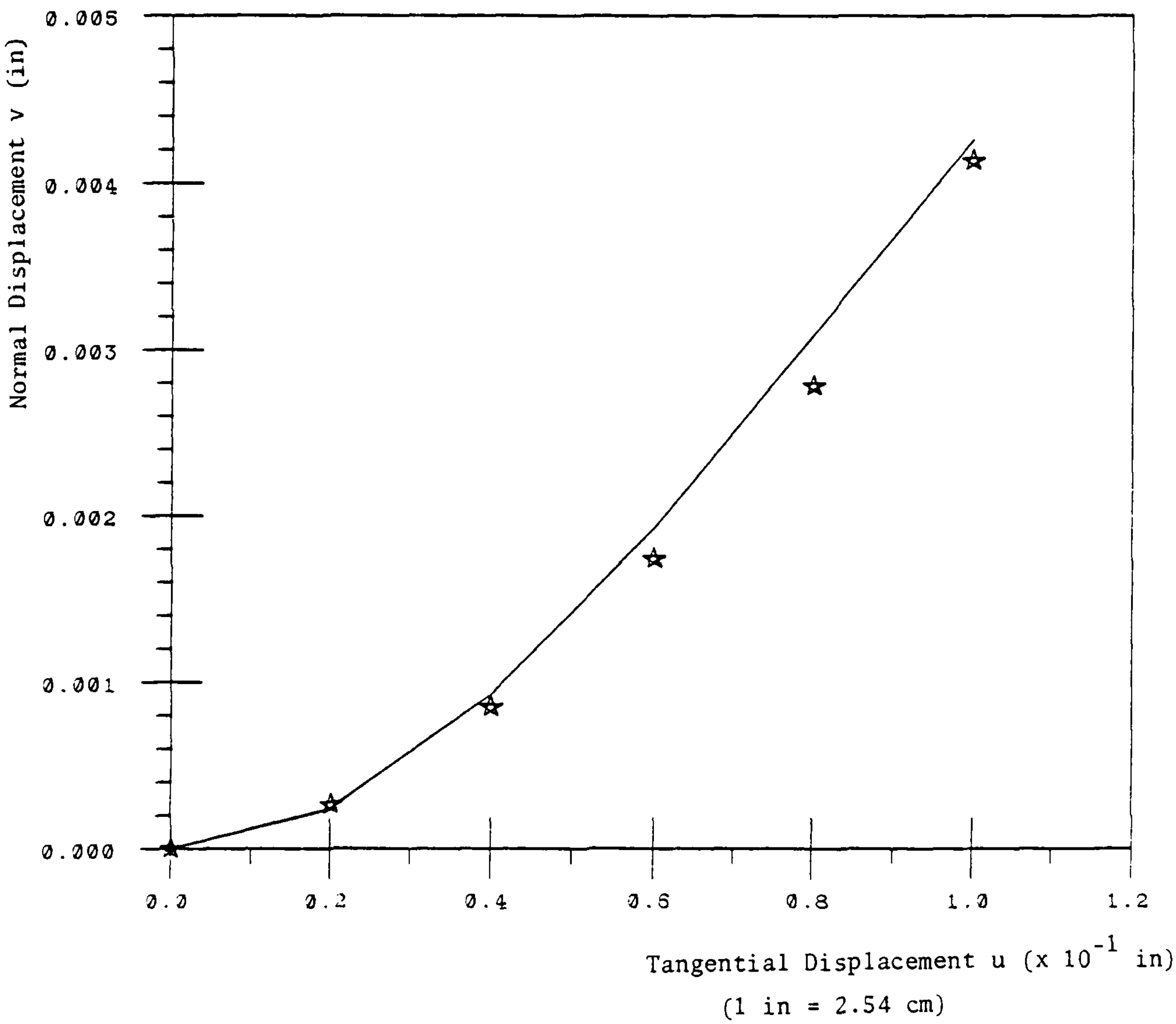

Fig. 21 Comparisons Between Back Predictions and Observations for Dilation and Tangential Displacement Relation in Shear Test (CNS) (Data Set #2)

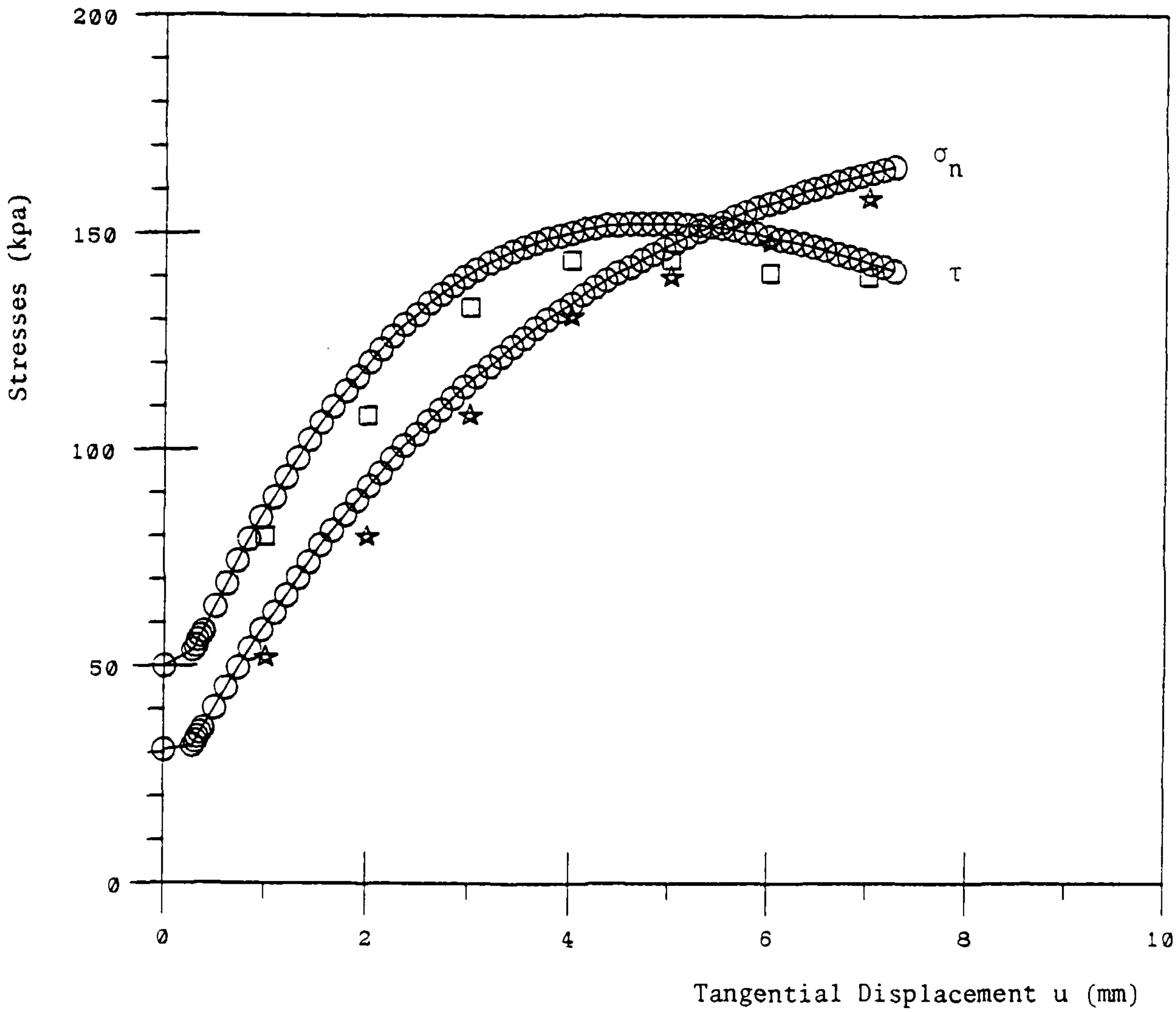

Fig. 22 Comparisons Between Back Predictions and Observations for Shear Stress, Normal Stress σ_n and Tangential Displacement Relation in Shear Test with Constant Normal Stiffness (CNK) (Data Set #3)

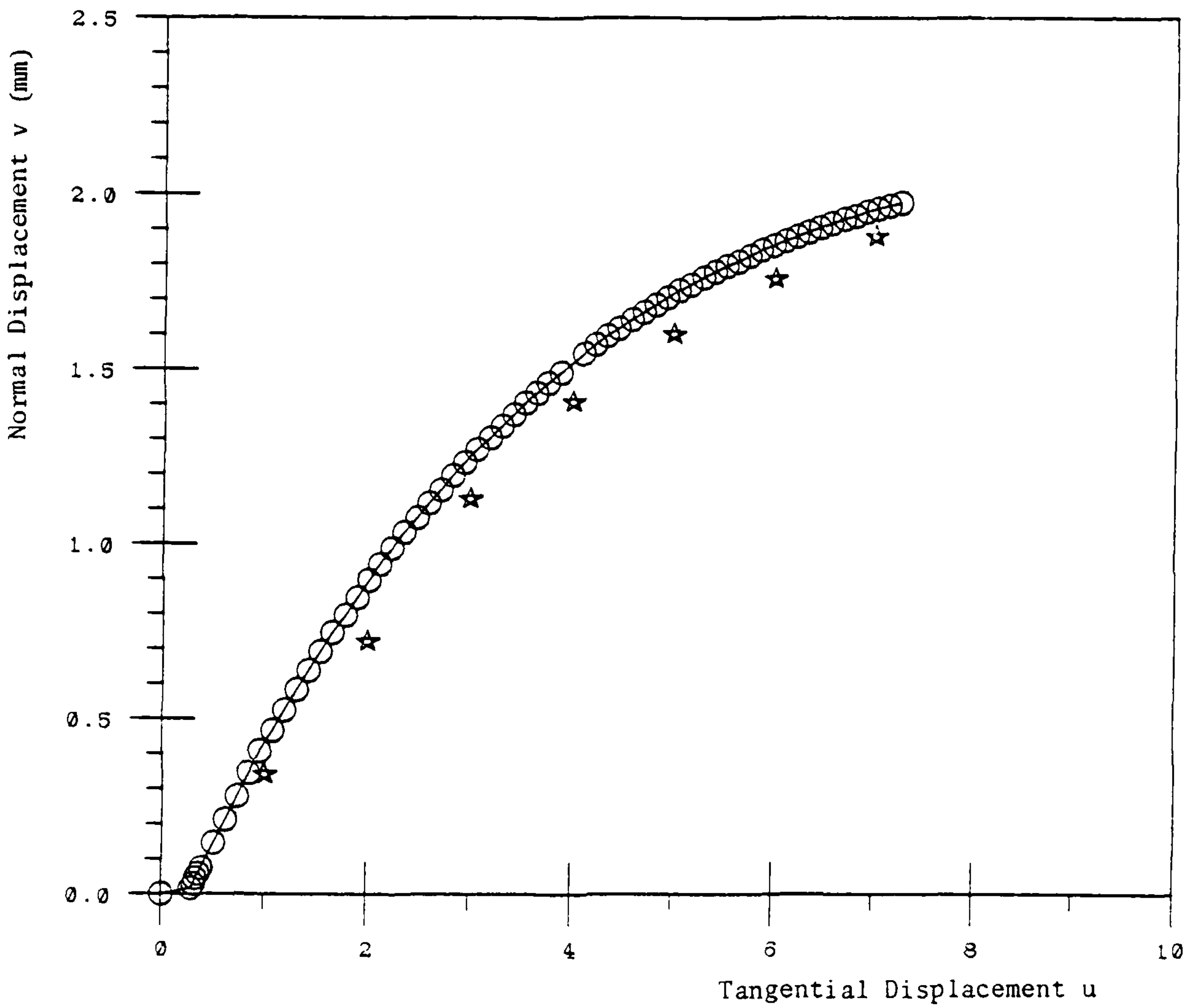

Fig. 23 Comparisons Between Back Predictions and Observations for Dilation and Tangential Displacement Relation in Shear Test (CNK) (Data Set #3)

theory is applied with modification. The major modification is made for the flow rule since the direction of displacement is determined mainly by the geometry of the joint, especially for very stiff joints.

Verification is made for three typical tests, the normal compression test (NCT), the shear test with constant normal stress (CNS) and the shear test with constant normal stiffness (CNK). The back predictions are very satisfactory.

REFERENCES

1. C. S. Desai, S. Somasundaram and G. Frantziskonis, "A Hierarchical Approach for Constitutive Modelling of Geologic Materials," Int. J. Num. Analyt. Meth. in Geomech., 10, No. 3, 1986.

2. C. S. Desai and K. L. Fishman, "Constitutive Models for Rocks and Discontinuities (Joints)," Proc., 28th U. S. Symp. Rock Mech., Univ. of Arizona, Tucson, AZ, pp. 609-619, 1987.

3. K. L. Fishman and C. S. Desai, "A Constitutive Model for Hardening Behavior of Rock Joints," Proc., 2nd Int. Conf. on Constitutive Laws for Engineering Materials, University of Arizona, Tucson, AZ, pp. 1043-1053.

4. K. L. Fishman, "Constitutive Modelling of Idealized Rock Joints under Quasi-Static and Cyclic Loading," Ph.D. Dissertation, Department of Civil Engineering and Engineering Mechanics, University of Arizona, 1987.

5. R. Yoshinaka and T. Yamabe, "Joint Stiffness and the Deformation Behavior of Discontinuous Rock," Int. J. Rock Mech., Vol. 23, No. 1, pp. 19-28, 1986.

6. D. M. Dight and H. K. Chiu, "Prediction of Shear Behavior of Joints Using Profiles," Int. J. Rock Mech., pp. 369-386, 1981.